Problem Set 3 Day 5 Questions

1. The local supermarket buys lettuce each day to ensure really fresh produce. Each morning any lettuce that is left from the previous day is sold to a dealer that resells to farmers who use it to feed their animals. This week the supermarket can buy fresh lettuce for $4.00 a box. The lettuce is sold for $10.00 a box and the dealer that sells old lettuce is willing to pay $1.50 a box. Past history says that tomorrow’s demand for lettuce averages 250 boxes with a standard deviation of 34 boxes. How many boxes of lettuce should the supermarket purchase tomorrow? (Assume that the demand is normally distributed.)

**ANSWER:**
We have $C_o = 4.00 - 1.50 = 2.50$ and $C_u = 10.00 - 4.00 = 6.00$. Thus, the newsvendor fractile is $p = C_u/(C_u + C_o) = 70.59\%$, and the corresponding $z$ value is $z = 0.54$. Then, the optimal quantity is $Q = 250 + 34 \cdot z = 268.4$.

2. On a given Vancouver-Montreal flight there are 200 seats. Suppose the ticket price is $475 for each seat, and the number of passengers who reserve a seat but do not show up for departure is normally distributed with mean 30 and standard deviation 15. You decide to overbook the flight and estimate that the average loss from a passenger who will have to be “bumped” (if the number of passengers exceeds the number of seats) is $800 (in addition to refunding the original ticket price). What is the maximum number of reservations that should be accepted?

**ANSWER:**
Clearly, we should accept at least 200 reservations. We perform a marginal cost analysis. Suppose that we sell $Q$ number of reservations, where $Q \geq 200$. The marginal benefit from selling the last reservation is $475, which you obtain only if there is enough seats for all passengers who showed up (which is the case when the number of no-shows exceeds $Q - 200$). Therefore, the marginal benefit is:

$$475 \cdot P[\text{Number of No-Show} > (Q - 200)] = 475 \cdot (1 - P[\text{Number of No-Show} \leq (Q - 200)])$$

To compute the marginal benefit, note that the potential cost is to pay for the “bumped” passenger ($800), and this event happens if the total number of customers who showed up is at most 200 (which is the case when the number of no-shows does not exceed $Q - 200$). Thus, the marginal cost is, on average,

$$800 \cdot P[\text{Number of No-Show} \leq (Q - 200)]$$

Equating the marginal benefit and the marginal cost

$$475 \cdot (1 - P[\text{Number of No-Show} \leq (Q - 200)]) = 800 \cdot P[\text{Number of No-Show} \leq (Q - 200)]$$
\( P[\text{Number of No-Shows} \leq (Q - 200)] = \frac{475}{(800 + 475)} = 0.3725. \)

Then, the z value should be \( z = -0.32 \), and thus we obtain that

\( (Q - 200) = 30 + (-0.32)(15) = 25.2. \)

Thus, the maximum number of reservations to accept is \( 200 + 25.2 = 225.2 \)

**Alternative way of thinking about this solution.** The overage cost is $800 (overbooking means that a passenger has to be “bumped”, which costs $800. The underage cost is $475 (corresponding to an empty seat). The critical ratio is \( \frac{475}{(800 + 475)} = 0.3725. \) The \( z \)-value associated with this critical ratio is \( z = -0.32 \). Thus, the optimal number of overbooking is \( 30 + (-0.32)(15) = 25.2. \) Thus, the maximum number of reservations to accept is \( 200 + 25.2 = 225.2 \) or 252.

3. The publisher of the Vancouver Sun incurs $0.20 for each copy of newspaper it prints, and charges $0.75 to Safeway for each copy that Safeway purchases. Readers pay $1 to Safeway for each copy of the Vancouver Sun. Thus, the publisher and Safeway constitute a twotier supply chain. Suppose the daily demand for the Vancouver Sun at a Safeway store is normally distributed with mean 100 and standard deviation 30.

(a) How many newspapers should Safeway purchase to maximize its own profit?
    (b) How many newspapers should Safeway keep so that the supply chains profit is maximized?

**ANSWER:**
(a) For Safeway, \( C_o = 0.75 \) and \( C_u = 1.00 - 0.75 = 0.25. \) Thus, the newsvendor fractile is \( p = \frac{C_u}{(C_u + C_o)} = 25\% \), and the corresponding \( z \)-value is \( z = 0. -0.67. \) Then, the optimal quantity is \( Q = 100 + 30 \cdot z = 79.8 \) to maximize Safeway’s own profit.

(b) Now, consider the supply chain consisting of both Safeway and the publisher. Each paper that is not sold incurs this supply chain the cost of \( C_o = 0.20. \) Each paper that is sold earns the supply chain the profit of \( C_u = 1.00 - 0.20 = 0.80. \) Thus, \( z = 0.84 \) and the optimal quantity is \( 100 + 30 \cdot z = 125.2 \) to maximize the supply chain’s profit.