Lecture Outline

• Fleet Assignment Problem
  ▪ Objectives and principal economic trade-offs

• Single Flight Leg vs. Network Fleet Assignment
  ▪ Operational constraints and modeling assumptions
  ▪ Example: Network fleet assignment problem

• Network Fleet Assignment Optimization
  ▪ Objective function and constraints
  ▪ Solution times

• Coldstart: Fleet Assignment at Delta Air Lines
SCHEDULE DEVELOPMENT

• Involves several interrelated decisions, which to date have not been fully integrated:

  Frequency Planning: Number of departures to be offered on each route, non-stop versus multi-stop

  Timetable Development: Flight departure and arrival times, including connections at airline hubs

  Fleet Assignment: Aircraft type for each flight, based on demand and operating cost estimates

  Aircraft Rotation Planning: Links consecutive flights to ensure balanced aircraft flows on the network.
Fleet Assignment Problem

- Given a schedule of flight legs (origin, destination, departure and arrival times), as well as:
  - Number of Aircraft by Equipment Type
  - Turn Times by Fleet Type at each Station
  - Other Restrictions: Maintenance, Gate, Noise, Runway, etc.

- Operating Costs and Spill Costs determine the Total Potential Contribution of each Flight, by Fleet Type

- What is the optimal (contribution/profit maximizing) assignment of aircraft to each flight leg?
Trade-offs in Fleet Assignment

• Operating costs increase with size of airplane for any given flight (typically)
  ▪ Larger aircraft have higher ownership and maintenance costs
  ▪ Increased fuel burn with greater capacity and weight
  ▪ More (and perhaps higher paid) crew members required

• Spill costs decrease with size of airplane
  ▪ SPILL is rejected demand due to inadequate capacity
  ▪ Larger aircraft accommodate more demand and generate more revenue, meaning less spill and lower spill costs

• Economic trade-off in choosing optimal fleet type
  ▪ Too large an aircraft leads to higher costs, empty seats
  ▪ Too small an aircraft leads to higher load factors but more rejected demand and lost revenue potential
Fleet Assignment Example – Single Leg

Demand = 100  
Fare = $100

<table>
<thead>
<tr>
<th>Fleet Type</th>
<th>Capacity</th>
<th>Spill Cost</th>
<th>Op. Cost</th>
<th>Assignment Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>80</td>
<td>$2,000</td>
<td>$5,000</td>
<td>$7,000</td>
</tr>
<tr>
<td>ii</td>
<td>100</td>
<td>$0</td>
<td>$6,000</td>
<td>$6,000</td>
</tr>
<tr>
<td>iii</td>
<td>120</td>
<td>$0</td>
<td>$7,000</td>
<td>$7,000</td>
</tr>
<tr>
<td>iv</td>
<td>150</td>
<td>$0</td>
<td>$8,000</td>
<td>$8,000</td>
</tr>
</tbody>
</table>
Single Leg vs. Network Fleet Assignment

- Single leg spill and fleet assignment is unrealistic:
  - Larger aircraft must be available at origin airport at required departure time
  - Larger aircraft must return (or continue onward) from destination airport
  - Smaller aircraft must be assigned to an alternative profitable flight leg
  - Crew rotations, maintenance and other considerations

- Also, single leg model assumes 100% local traffic
  - Changing aircraft size on one flight will affect connecting network passenger flows on other flights
  - No recapture assumed – changing aircraft size on one flight will affect passenger loads on other flights on the same route
Example: Network Fleet Assignment

Flight Network

- **ORD** (Chicago O’Hare)
- **LGA** (New York LaGuardia)
- **BOS** (Boston Logan)

Connections:
- CL50x (2 flights) from ORD to LGA
- CL50x (3 flights) from LGA to BOS
- CL33x (3 flights) from LGA to BOS
- CL30x (3 flights) from BOS to LGA
- CL55x (2 flights) from ORD to LGA
**Example: Network Fleet Assignment**

## Flight Schedule, Fares, & Demand

<table>
<thead>
<tr>
<th>Flight #</th>
<th>From</th>
<th>To</th>
<th>Dept Time (EST)</th>
<th>Arr Time (EST)</th>
<th>Fare [$]</th>
<th>Demand [passengers]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CL301</td>
<td>LGA</td>
<td>BOS</td>
<td>1000</td>
<td>1100</td>
<td>150</td>
<td>250</td>
</tr>
<tr>
<td>CL302</td>
<td>LGA</td>
<td>BOS</td>
<td>1100</td>
<td>1200</td>
<td>150</td>
<td>250</td>
</tr>
<tr>
<td>CL303</td>
<td>LGA</td>
<td>BOS</td>
<td>1800</td>
<td>1900</td>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>CL331</td>
<td>BOS</td>
<td>LGA</td>
<td>0700</td>
<td>0800</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>CL332</td>
<td>BOS</td>
<td>LGA</td>
<td>1030</td>
<td>1130</td>
<td>150</td>
<td>300</td>
</tr>
<tr>
<td>CL333</td>
<td>BOS</td>
<td>LGA</td>
<td>1800</td>
<td>1900</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>CL501</td>
<td>LGA</td>
<td>ORD</td>
<td>1100</td>
<td>1400</td>
<td>400</td>
<td>150</td>
</tr>
<tr>
<td>CL502</td>
<td>LGA</td>
<td>ORD</td>
<td>1500</td>
<td>1800</td>
<td>400</td>
<td>200</td>
</tr>
<tr>
<td>CL551</td>
<td>ORD</td>
<td>LGA</td>
<td>0700</td>
<td>1000</td>
<td>400</td>
<td>200</td>
</tr>
<tr>
<td>CL552</td>
<td>ORD</td>
<td>LGA</td>
<td>0830</td>
<td>1130</td>
<td>400</td>
<td>150</td>
</tr>
</tbody>
</table>
## Fleet Information

<table>
<thead>
<tr>
<th>Fleet type</th>
<th>Number of aircraft owned</th>
<th>Capacity [seats]</th>
<th>Per flight operating cost [$000]</th>
<th>LGA - BOS</th>
<th>LGA – ORD</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC-9</td>
<td>1</td>
<td>120</td>
<td></td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>B737</td>
<td>2</td>
<td>150</td>
<td></td>
<td>12</td>
<td>17</td>
</tr>
<tr>
<td>A300</td>
<td>2</td>
<td>250</td>
<td></td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>
**Example: Network Fleet Assignment**

**Evaluating assignment profits…**

*Profitability [$000 per day]*

<table>
<thead>
<tr>
<th>Flight #</th>
<th>DC-9</th>
<th>B737</th>
<th>A300</th>
</tr>
</thead>
<tbody>
<tr>
<td>CL301</td>
<td>8</td>
<td>10.5</td>
<td>22.5</td>
</tr>
<tr>
<td>CL302</td>
<td>8</td>
<td>10.5</td>
<td>22.5</td>
</tr>
<tr>
<td>CL303</td>
<td>5</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>CL331</td>
<td>8</td>
<td>10.5</td>
<td>7.5</td>
</tr>
<tr>
<td>CL332</td>
<td>8</td>
<td>10.5</td>
<td>22.5</td>
</tr>
<tr>
<td>CL333</td>
<td>8</td>
<td>10.5</td>
<td>7.5</td>
</tr>
<tr>
<td>CL501</td>
<td>33</td>
<td>43</td>
<td>40</td>
</tr>
<tr>
<td>CL502</td>
<td>33</td>
<td>43</td>
<td>60</td>
</tr>
<tr>
<td>CL551</td>
<td>33</td>
<td>43</td>
<td>60</td>
</tr>
<tr>
<td>CL552</td>
<td>33</td>
<td>43</td>
<td>40</td>
</tr>
</tbody>
</table>
Example: Network Fleet Assignment

Assign 1 DC9, 2 B737 and 2 A300 to Time-Line Network:
Fleet Assignment Solution

Assign 1 DC9, 2 B737 and 2 A300 to Time-Line Network:

Revenue = $428,500  Cost = $148,000  Profit = $280,500
Network Fleet Assignment

Objective Function

- For each fleet type/flight combination:

  Assignment Cost $\equiv$ Operating cost + Spill cost

- Operating cost of assigning a fleet type $k$ to a flight leg $j$ is relatively straightforward to compute
  - Can capture range restrictions, noise restrictions, water restrictions, etc. by assigning “infinite” costs

- Spill cost for flight leg $j$ and fleet assignment $k$
  - Average revenue per passenger on $j \times \text{MAX}(0, \text{unconstrained demand for } j - \text{number of seats on } k)$
  - But revenue for each flight leg is affected by fare class mix (RM) as well as itinerary mix in a network
Constraints

• Cover Constraints
  ▪ Each flight must be assigned to exactly one fleet type

• Balance Constraints
  ▪ Number of aircraft of a fleet type arriving at a station must equal the number of aircraft of that fleet type departing

• Aircraft Count Constraints
  ▪ Number of aircraft of a fleet type used cannot exceed the number available
Formulation: The Fleet Assignment Model (FAM)

maximize \( \text{PROFIT} \)

subject to

- Flight Cover
- Aircraft Balance
- Aircraft Count
- Integrality and Non-negativity

Solution

- Solve fleet assignment problems for large network carriers (10-14 fleets, 2000-3500 flights) within 10-20 minutes of computation time on workstation class computers

COLDSTART:
Fleet Assignment at Delta Air Lines

• 1994 Interfaces article describes implementation of large-scale network fleet assignment optimization
  ▪ 2500 domestic flight legs per day
  ▪ 450 aircraft of 10 different fleet types

• Mixed-integer linear program
  ▪ Minimize assignment costs over the Delta domestic network and schedule for one day
  ▪ Assigns fleet types to each leg, not tail numbers (aircraft routing performed subsequently)

• First OR application of this size implemented at Delta
  ▪ Use of this model estimated to increase operating profit by $100 million per year
Constraints and Issues in Coldstart

- Certain pairs of legs must be assigned same fleet
  - Provide one-stop, same-plane service through the hub
  - Tag-end flights that must be operated with same aircraft (e.g., IST-GRU-EZE)

- Model includes maintenance requirements
  - Use “maintenance arcs” to represent flights that must be covered with an aircraft
  - For example, a B757 must be at a designated base each night

- Crew considerations
  - Common fleet families use same pilot aggregates
  - Penalize fleet assignments that require extended crew rest periods – for example, when only 1 flight into/out of a city is assigned an aircraft type
Constraints and Issues in Coldstart

• Aircraft performance can differ within each fleet type
  ▪ Different engines; take-off and landing weights
  ▪ Not all aircraft equipped to serve over-water routes

• Airport characteristics and restrictions
  ▪ Runway lengths and temperature limitations
  ▪ Certain airports have noise restrictions and/or curfews for specific aircraft types

• Assumed turn times determine aircraft availability
  ▪ Minimum turn around times vary by both aircraft type and airport
  ▪ Larger aircraft require longer turn times
  ▪ International flights require longer turn times than domestic flights with same fleet type
Coldstart in the Scheduling Process

- **Changes to the functions of DL schedule planners**
  - Model performs assignments to schedule, while planner reviews and analyzes the impacts of assignment changes
  - Focus on cost and revenue inputs to the model instead of the actual optimization process

- **Model allows for what-if analysis by planners**
  - Compare results from two different optimization runs to compare impacts of various costs and constraints
  - Evaluate the changes required in order to change the aircraft assigned to a particular flight leg – up-line and down-line swaps required