Lecture Outline

• Terms and Definitions
  ▪ Demand, Load and Spill
  ▪ Airline Demand Variability

• Spill Analysis: Boeing Spill Model
  ▪ Estimating Spill Given Observed Load Factors
  ▪ Use of Spill Tables
  ▪ Impacts of Different Size Aircraft

• Applications to Cabin Configuration

• Spill and Recapture Across Multiple Flights

• Impacts of RM on Spill
Terms and Definitions

- **DEMAND**: Total number of potential passengers wishing to book a seat on a given flight leg
  - Total potential demand at current fare structure

- **LOAD**: Number of passengers actually carried
  - When demand is less than capacity, LOAD = DEMAND

- **SPILL**: Number of potential passengers unable to book a seat due to insufficient capacity
  - Also known as “rejected demand”
  - Equal to DEMAND minus LOAD
“Spill” vs. “Denied Boardings”

- **Spill** occurs when potential demand for a flight leg is greater than the physical capacity of the aircraft
  - Spill can occur whether or not the airline is using overbooking methods
  - For spill analysis, typically assume no overbooking or “perfect” overbooking in which no-shows are predicted correctly
  - Spill occurs during the pre-departure booking process

- **Denied Boardings** occur on overbooked flights when more passengers than capacity show up
  - Denied boardings occur because the airline overbooked too aggressively, not because the aircraft was too small
  - DBs occur at the gate just before departure
Airline Demand Variability

- **Total demand for a flight leg varies**
  - Cyclically: Season of year; day of week; time of day
  - Stochastically: Random fluctuations in demand

- **Total demand potential for a flight leg represented with a Gaussian distribution**
  - Mean and standard deviation over a schedule period
  - K-factor = coefficient of variation = \( \frac{\sigma}{\mu} \)

- **K-factor of total unconstrained demand**
  - Can vary by route, by schedule period
  - Higher for leisure markets and longer schedule periods
  - Typically assumed to range from 0.20 to 0.40

- **But, total unconstrained demand cannot be observed**
  - Unless aircraft capacity is always too large for demand
Example: Individual Flight Departures

<table>
<thead>
<tr>
<th>DATE</th>
<th>LOAD</th>
<th>CAP</th>
<th>LF</th>
<th>SPILL?</th>
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<td>01 APR</td>
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<td>125</td>
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<tr>
<td>08 APR</td>
<td>125</td>
<td>125</td>
<td>100%</td>
<td>LIKELY</td>
</tr>
<tr>
<td>15 APR</td>
<td>108</td>
<td>125</td>
<td>86%</td>
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<td>22 APR</td>
<td>83</td>
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</tr>
<tr>
<td>29 APR</td>
<td>123</td>
<td>125</td>
<td>98%</td>
<td>POSSIBLY</td>
</tr>
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</table>

- Sample of n=5 flight departures with ALF=85.0% given capacity 125 seats – spill occurred in 2/5 cases.
Frequency Histogram of Flight Loads

Source: Boeing (1978)
Demand with Mean=125, Sigma=45

Spill (rejected demand and lost revenue) is reduced with larger capacity.
Spill Analysis: Boeing Spill Model

• **Objective:** Estimate actual “unconstrained” demand for a sample of flights where spill has occurred.

• **Observations:** Sample of flight leg loads (constrained) over a representative time period:
  - Perhaps adjusted for future seasonality and/or traffic growth

• **Assumptions:**
  - Unconstrained demand for a series of flight departures can be represented by a Gaussian distribution
  - We use observed Average Load Factor and an **ASSUMED** k-factor to estimate unconstrained demand

• **Boeing Spill Tables can be used to minimize calculations**
Example: Sample of Flight Departures

- Mean load = 106.2 passengers (85.0% LF) with observed standard deviation = 18.6
  - But, observed sigma constrained by capacity
  - Both mean and sigma are therefore smaller than actual demand
- Assume $K=0.35$ for unconstrained demand
  - Based on “market knowledge” and expected demand variability during schedule period under consideration
- Spill Table ($K=0.35$) shows relationships between
  - AVERAGE LOAD FACTOR = Mean Load/Capacity
  - DEMAND FACTOR = Mean Demand/Capacity
  - SPILL FACTOR = Mean Spill/Capacity
- “Spill Rate” = Mean Spill / Mean Demand
  - Historical target for spill rate is 5-10% or less
Spill Table for $K=0.35$

DF and SF given LOAD FACTOR

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<tr>
<th>LF</th>
<th>DF</th>
<th>SF</th>
<th>LF</th>
<th>DF</th>
<th>SF</th>
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<td>0.716</td>
<td>0.016</td>
<td>0.705</td>
<td>0.722</td>
<td>0.017</td>
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<tr>
<td>0.710</td>
<td>0.729</td>
<td>0.019</td>
<td>0.715</td>
<td>0.735</td>
<td>0.020</td>
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<td>0.720</td>
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<td>0.022</td>
<td>0.725</td>
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<td>0.047</td>
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<tr>
<td>0.780</td>
<td>0.830</td>
<td>0.050</td>
<td>0.785</td>
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<td>0.054</td>
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<tr>
<td>0.790</td>
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<tr>
<td>0.800</td>
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<td>0.810</td>
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<td>0.860</td>
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<td>0.870</td>
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<td>0.875</td>
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<td>0.880</td>
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<td>0.205</td>
<td>0.895</td>
<td>1.115</td>
<td>0.220</td>
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</table>

- Assuming underlying demand has $K=0.35$
- Then, 0.850 observed average load factor translates to 0.972 demand factor and 0.122 spill factor
- Load factor = demand factor – spill factor

Source: Boeing
Spill Table Calculations

• Given observed LF and assumed K=0.35
  - DF = 0.972 from Table, and SF = 0.122
  - [Note that DF = LF + SF, always!]

• We can now calculate the following estimates:
  - Mean total demand = DF * Capacity = 0.972*125= 121.5
  - Std deviation of Demand = 0.35 * 121.5 = 42.5
  - Mean spill per departure = SF * Capacity = 0.122*125 = 15.3
    [NOTE also: Mean Spill = Mean Demand – Mean Load]
  - Spill Rate = Mean Spill/Mean Demand = 15.3 / 121.5 = 12.6%
Impact of Larger Capacity (140 seats)

- With estimated Mean Demand = 121.5 and Cap=140
  - Demand Factor = 121.5/140 = 0.868
  - [Mean Demand does not change with a change in capacity!]

- From Spill Table (K=0.35), with DF=0.868
  - New average LF expected to be 0.802 (with some interpolation)
  - New mean load = 0.802 * 140 = 112.3 passengers, an increase of 6.1 passengers per departure
  - New average spill = 0.066*140 = 9.2 passengers, a decrease of 6.1 passengers per departure
  - New spill rate = 9.2/121.5 = 7.6%

- Use of larger aircraft increases load, reduces spill, but decreases load factor. Demand does not change.
### Spill Table for K=0.35

**LF and SF given DEMAND FACTOR**

<table>
<thead>
<tr>
<th>DF</th>
<th>LF</th>
<th>SF</th>
<th>DF</th>
<th>LF</th>
<th>SF</th>
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</table>

- Assuming underlying demand has K=0.35
- Then, 0.870 estimated demand factor translates to 0.803 average load factor and 0.067 spill factor
- Demand factor = load factor + spill factor

*Source: Boeing*
DF vs. LF for Demand (K=0.35)
Alternative Aircraft Capacities

- Should the airline operate a 140-seat aircraft to serve this demand distribution?

- Increasing capacity by 15 seats expected to increase average load per departure by 6.1 passengers
  - Increase in revenue per flight = 6.1 passengers * average fare

- But, changing this fleet assignment to a larger aircraft will increase operating costs as well
  - Increase in operating costs = difference in cost/block-hour * number of block-hours for flight leg in question
Applications to Cabin Configuration

• Additional seats in Premium Class reduce premium spill and increase revenues; but reduction in Economy seats increases economy spill and reduces economy revenue

• Spill model can be used to estimate the trade-off in premium revenue gain vs. economy revenue loss
Cabin Configurations for B767-300

18 first
60-in pitch

46 business
38-in pitch

154 premium
32-in pitch

218 passengers

24 first
38-in pitch

245 premium
32-in pitch

269 passengers

32-in pitch

286 passengers

Source: Boeing Commercial Airplanes
Spill and Recapture Across Multiple Flights

Airline 1’s flights:
- have a certain number of passengers getting their first choice on their desired flight
- have some number getting recaptured on other flights
- attract some passengers from Airline 2 (spill-in)
- have some passengers sell-up to get on their desired flight
- lose some number of passengers either to Airline 2 (spill-out) or through No Go

Source: Abramovich (2013)
Reduced Flight 1 Capacity

With reduced capacity on Flight 1, we would expect:

- a decreased number of passengers to get their first choice on Flight 1
- increased recapture on other flights
- to attract some passengers from Airline 2 (spill-in)
- more passengers to sell-up
- to lose more passengers either to Airline 2 (spill-out) or through No Go, given fewer available seats

Source: Abramovich (2013)
Increase Flight 1 Capacity

Flight 1  Flight 2  Flight 3

With increased capacity on Flight 1, we would expect:

- low LFs on Flight 1 and some changes in LFs on other flights
- an increased number of passengers to get their first choice on Flight 1
- decreased recapture on all flights as more passengers get their first choices
- to attract some passengers from Airline 2 (spill-in)
- virtually no sell-up due to the largely increased capacity
- to lose fewer passengers either to Airline 2 (spill-out) or through No Go, given far more available seats

Source: Abramovich (2013)
**RM Systems Reject Demand**

- Revenue management system generates booking limits for each class to maximize revenue
  - Protect seats for high fare passengers, reject low-fare bookings when demand factor is high

<table>
<thead>
<tr>
<th>CABIN CAPACITY =</th>
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</tr>
</thead>
<tbody>
<tr>
<td>AVAILABLE SEATS  =</td>
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</tr>
</tbody>
</table>

<table>
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<tr>
<th>BOOKING CLASS</th>
<th>AVERAGE FARE</th>
<th>SEATS BOOKED</th>
<th>FORECAST DEMAND MEAN</th>
<th>SIGMA</th>
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</table>

*Source: Abramovich (2013)*
**Impacts of RM on Marginal Revenue**

- Marginal revenue per additional seat decreases with increasing capacity.
- Most additional bookings are in lower classes.

*Standard Leg RM*

*Fare Class Mix*  
*Marginal Revenue*

*Source: Abramovich (2013)*